

AN EXPERIMENTAL STUDY ON PERFORMANCE OF SHAPE MEMORY ALLOY SUSPENSION PENDULUM DAMPING SYSTEM

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ABSTRACT

A systematic study is conducted on variation regularities for such properties of SMA wires with the diameter of wires, strain amplitude and loading cycles as the stress-strain curve, characteristic stresses, energy dissipation capacity, equivalent damping ratio. The results show: The diameter increase of SMA wires can degrade the mechanical properties of SMA wires; SMA wires can exhibit a good and stable hysteretic performance when the strain amplitude is 6% and the loading cycle is 15. Utilizing the superelasticity of SMA and combining the operating principle of the tuned mass damper, a new kind of SMA suspension pendulum damping system, which is easy to disassemble, was designed and fabricated with the trained SMA wires. Corresponding experiments were conducted to analyse the natural frequency of the damping system, phase relations between the mass vibrators and the controlled structure, variation regularity of the equivalent damping force with the mass vibrators and length of pendulum rod. The results indicate that phase relations between the mass vibrators and the controlled structure can desirably fall within $150^{\circ}\sim 180^{\circ}$ when this damping system subjected to the sine waves and real earthquake waves. Meanwhile, the equivalent damping force increases significantly with the amplitude of external loads. In conclusion, this kind of damping system can provide a stable and efficient damping force and be simply applied to structural vibration control, thus to protect structures free from strong dynamic disasters.

KEYWORDS

Superelasticity, SMA suspension pendulum damping system, Phase relation, Equivalent damping force

Introduction

An energy dissipation device is an effective measure to realize the passive control of structure and reduce the vibration damage of structures under the earthquake and strong wind. At present the traditional passive energy dissipation devices are made of rubber, viscous liquid, visco-elastic material and low yielding point metal[1], which has many disadvantages such as material aging, poor long-term reliability, and irreversible residual deformation in the practical engineering application[2]. At the same time, those devices also cannot satisfy the demand of existing buildings for the simple installation and obvious protective effect (Urgent protection of building structures such as the ancient towers). So their applications are limited to an extent.

Shape memory alloy (Shape Memory Alloy, referred to as SMA) as a new intelligent material, has the characteristics of unique shape memory effect, superelasticity and high damping[3], thus which has been widely focused in the field of structural vibration control. Many scholars at home and abroad have carried out relevant researches on the basic material behaviours of SMA and its application in structural vibration control. Corb et al.[4] compared the control effect of SMA Cable and elastic plastic cable on the elasto-plastic vibration response of a single story frame structure and pointed out that the SMA Cable cannot only restrain the vibration of the structure but also give a good self-centering function. Chen Yun et al[5] proposed a type of energy dissipation enhanced SMA damper, which has a full hysteresis loop, a high energy dissipating capacity and presents a better energy dissipation effect than SMA stay cable. Ren Wen Jie, Li Hong nan et al[6] proposed a new self-centering SMA damper. The theoretical analysis was carried out on the frame structure on which the damper was installed; the results show: the damper can effectively restrain the displacement of the structure, the inter-story displacement and the residual displacement, but it will increase the acceleration of the structure.

Based on the theory of super elasticity of SMA, through the systematic studies on the variation regularity of SMA stress-strain curve and energy dissipation capacity with diameter of wires, strain amplitude and loading cycles, the suitable SMA material is determined for damper development. Combined with the basic working principle of the tuned mass damper, a new kind of SMA composite suspension damping system is designed and manufactured. This new damping system can be of easy disassembly. By the parametric variation of the pendulum mass and the pendulum length, the performance test of the damping system is carried out, and the phase relationship between the mass oscillator and the controlled structure is analysed as well as the variation law of the equivalent damping force. The effectiveness of the application of this damping system is verified in the structural vibration control.

Mechanical Properties of SMA Wire

Testing situation

The mechanical properties of 12 different working conditions were tested on SMA wires in this paper. A systematic study is conducted on variation regularity of stress-strain curve, characteristic stresses, energy dissipation capacity, equivalent damping ratio of SMA wires with the diameter of wires, strain amplitude and loading cycles. The experimental conditions are shown in Table 1.

Tab. 1 - Experimental conditions for SMA wire superelasticity

Test number	Specimen gauge length (mm)	Diameter (mm)	Strain amplitude (%)	Cycles
1	33.5	0.5	3	30
2			6	
3			8	
4	33.5	0.8	3	30
5			6	
6			8	
7	33.5	1.0	3	30
8			6	
9			8	
10	33.5	1.2	3	30
11			6	
12			8	

The SMA wire for the experiment is provided by The Saite Metal Materials Development Co Ltd of Northwest Nonferrous Metals Research Institute. The chemical composition of the SMA wire is Ti-50.8 at.% Ni. The specimen is 300mm in length, with an effective length of 100mm. Its phase transition temperature is M_f -42°C, M_s -38°C, A_f -13°C, and A_s -9°C. This experiment was carried out in the laboratory of materials science, Xi'an University of Science and Technology. The experiment adopted the HT-2402 computer servo control material testing machine with its maximum tension load of 100t and its load precision of $\pm 5\%$. The axial deformation was measured by means of displacement extensometer with a standard distance of 33.5mm. To ensure the accuracy of the initial test, prior to the start of each condition, the pre-tension of 10MPa ~ 30MPa was imposed upon the specimen so that the specimen could be stretched tightly. Loading / unloading protocol adopted a constant rate of loading / unloading; the loading was terminated until the strain of the wire reaching its preset strain amplitude, whereas unloading terminated till the axial force on the wire was less than 5N. Meanwhile, each test case was loaded for 30 cycles.

Test analysis

The single-cycle stress-strain curves of SMA wire loaded on austenite of the SMA wire at normal temperature are shown in Figure 1. The characteristic point a refers to the starting point of the platform during the loading branch; beyond the loading platform, the characteristic point b refers to the point which has an obvious mutation on the loading curve; the characteristic point c refers to the point where the stress-strain linear relationship begin to deviate during the unloading branch; the characteristic point d refers to the where the stress-strain curve begins to approximately proportionally fall [7,8].

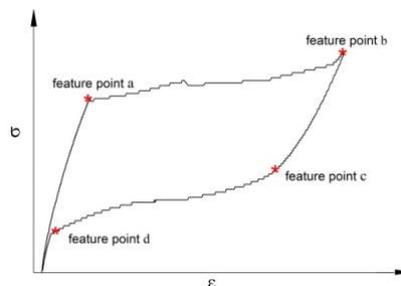


Fig. 1 – Characteristic points of the constitutive curve

Influence of wire diameter

In order to study the influence of material diameter on the mechanical properties of SMA wire, the diameters of 0.5mm, 0.8mm, 1.0mm, 1.2mm SMA wire were selected, and the loading strain amplitude was 6%. The results obtained are shown in Figure 2 and Table 2. With the increase of the diameter of the material, the stress-strain curve of SMA wire tends to be smooth, the accumulated residual deformation of the material increases, and the stress of each characteristic point is reduced to varying degrees. With the increase of diameter from 0.5mm to 1.2mm, the stress of the characteristic points a, b, c and d respectively decreased by 27.81%, 20.74%, 25.21% and 65.27%. When the diameter of the material is less than 0.8mm, the energy dissipation capacity and equivalent damping ratio of SMA wire have slight variations; when the diameter of the material is more than 0.8mm, with the increase of diameter, both behaviours were significantly decreased; the energy dissipation capacity is reduced by 21.19% and equivalent damping ratio decreased by 22.96%.

Tab. 2 - Mechanical properties of SMA wire with different diameters

Diameter/mm	σ_a /MPa	σ_b /MPa	σ_c /MPa	σ_d /MPa	ΔW /MJ.m ⁻³	ζ_a /%
0.5	483.83	585.69	331.04	203.72	12.43	6.49
0.8	447.62	527.20	358.10	139.26	12.22	6.01
1.0	420.17	502.93	331.94	118.23	10.52	5.34
1.2	349.26	464.20	247.57	70.74	9.63	5.00

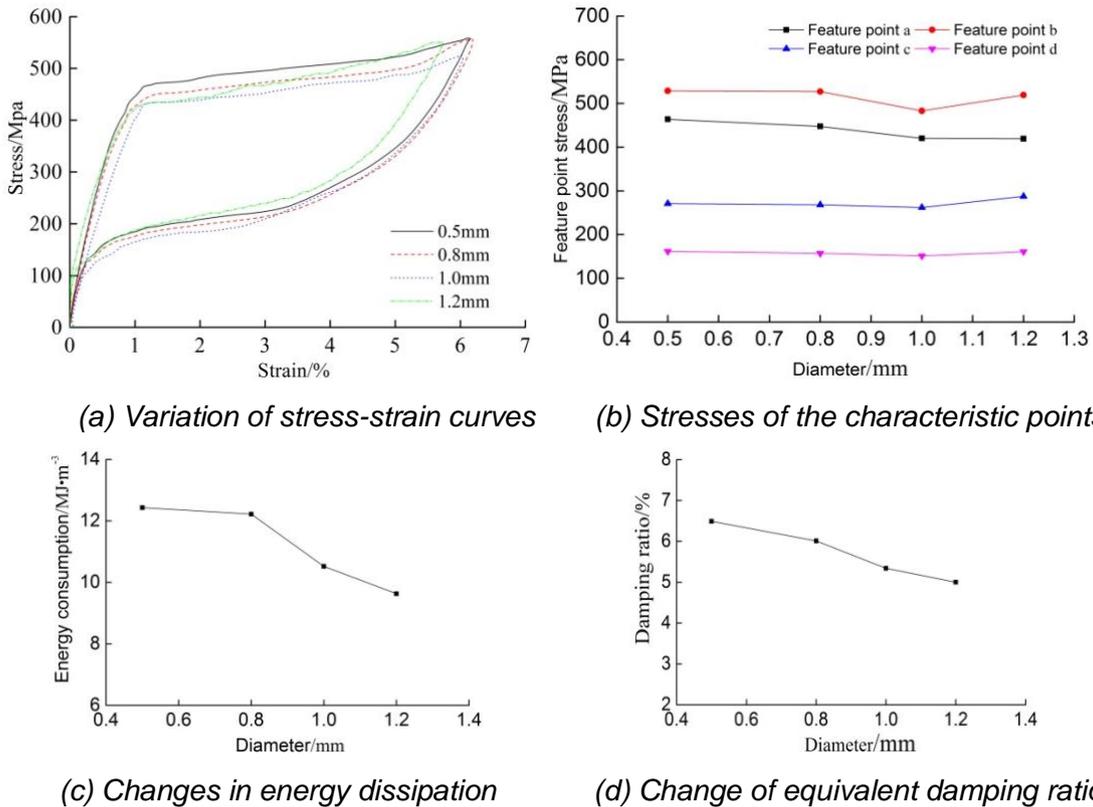


Fig. 2 – Effect of material diameter on the mechanical properties of austenite SMA wire

Effect of strain amplitude

The SMA wire with a diameter of 1.0 mm and the loading rate of 10mm/min were selected to analyse the influence of different strain amplitudes on the mechanical properties of SMA wire. The results are shown in Figure 3 and Table 3. With increasing the strain amplitude of SMA wire, the variation in the stress of characteristic points a, b, c is negligible. However, the stress of characteristic point d tends to decrease, which indicates that with the increase of the strain amplitude, the stress-strain curves of SMA tend to be full, and the energy dissipation capacity increases. When the strain amplitude is increased from 3% to 8%, the single cycle energy dissipation of SMA wire increases from 4.46 MJ.m⁻³ to 20.76 MJ.m⁻³, and the energy dissipation capacity is increased by nearly 4.7 times. When the strain amplitude is less than 6%, the equivalent damping ratio increases significantly; on the contrary, the damping ratio presents an inconsiderable change, which reveals that: although the absolute energy dissipation capacity of SMA wire increases with the increase of strain amplitude, the energy dissipation efficiency is most optimal when the strain amplitude is about 6%.

Tab. 3 - Mechanical Properties of austenite SMA wire at different strain amplitudes

Strain amplitude	σ_a /MPa	σ_b /MPa	σ_c /MPa	σ_d /MPa	ΔW /MJ.m ⁻³	ζ_a /%
3%	426.90	496.56	260.65	120.96	4.46	4.18
6%	420.17	509.30	254.65	101.86	12.70	6.09
8%	432.90	515.66	254.65	70.03	20.76	6.60

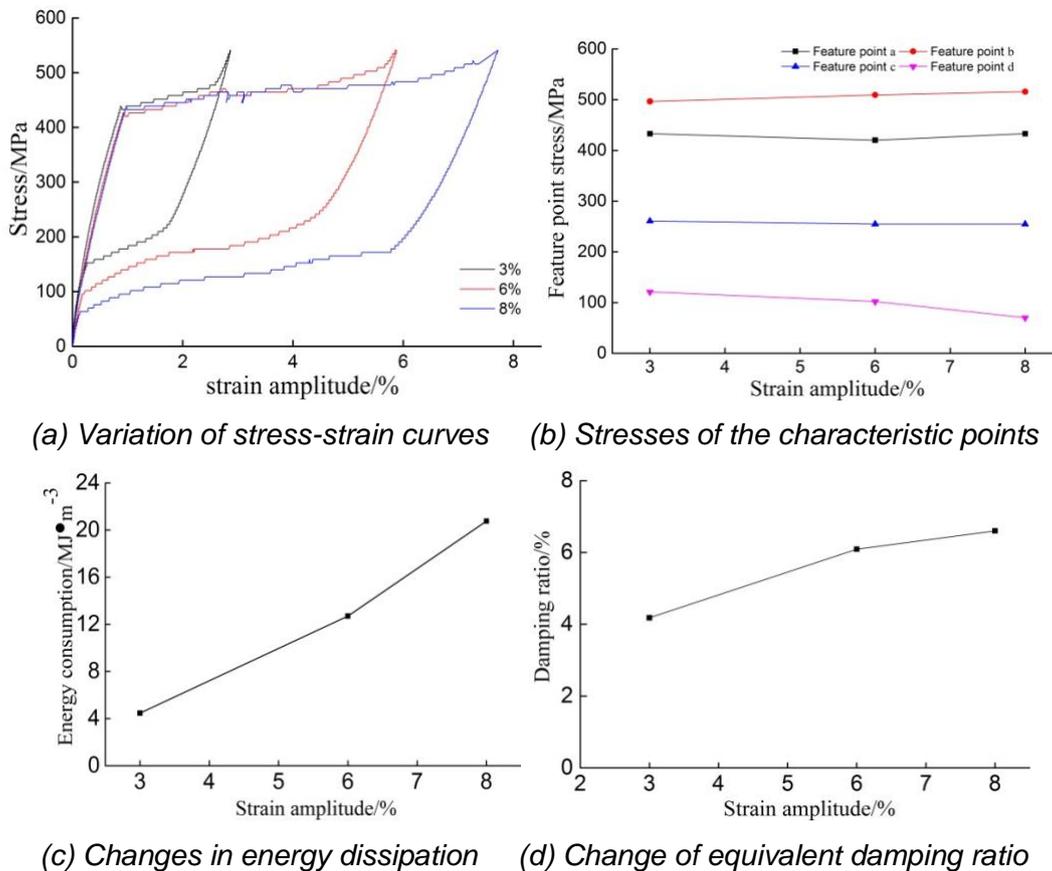


Fig. 3 – Effect of strain amplitude on the mechanical properties of austenite SMA wire

Effect of loading cycles

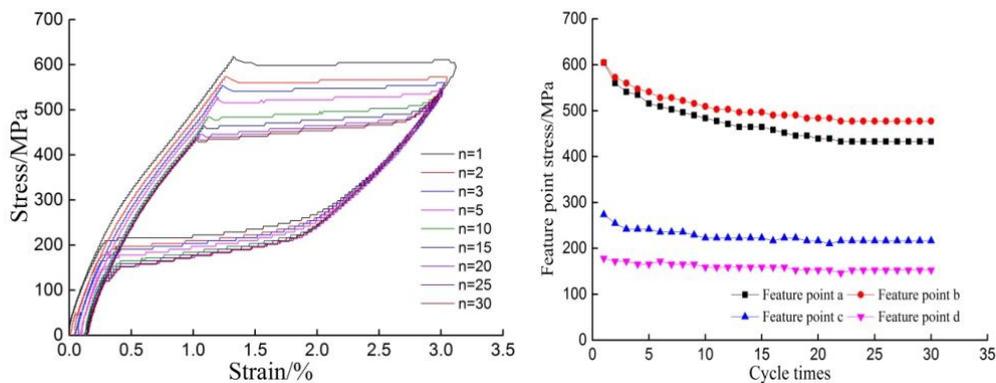
The SMA wire with a diameter of 0.5mm and strain amplitude of 6% was taken to analyse the influence of different loading cycles on the mechanical properties. The results are shown in Figure 4 and Table 4. With increasing the loading cycles, the stress-strain curve of SMA wire gradually becomes smoother, the accumulated residual deformation increases gradually, the residual deformation of each cycle decreases, the residual deformation of the 16th cycle is only 0.003%. In view of martensitic transformation, the stress of the characteristic points a is reduced respectively by 140.06MPa and by 171.89MPa after 15 cycles and 30 cycles. The stress decreases due to the first 15 cycles occupied 81.49%. The stress drop of the characteristic point b mainly occurred in the first 15 cycles. For the austenite transformation, the stress of the characteristic points c, d respectively decreased by 57.30MPa and 25.46MPa after 30 cycles, which respectively corresponded to the decreasing amplitude of 20.93% and 14.28%. The stress declines during the first 10 cycles respectively reached 88.89% and 75.02% of the total decreasing amplitudes, but the stress of characteristic points c and d tends to be stable after 10 cycles.

Meanwhile, the single cycle energy dissipation and the equivalent damping ratio of SMA wire gradually decrease with the increase of the number of cycles. After 30 cycles, the single cycle energy dissipation is reduced by 2.405 MJ.m⁻³, and the decreasing amplitude reached 35.16%. The equivalent damping ratio is reduced by 1.95%, and its decreasing amplitude was 31.91% after 30 cycles. In the early stage, the cyclic energy dissipation and the equivalent damping ratio decrease rapidly, and the energy dissipation capacity and the equivalent damping tend to be stable after 15 cycles.

It can be seen that the mechanical properties of austenite SMA wire are greatly influenced by the number of cycles. Therefore, for utilizing the desirable superelastic properties of SMA material in the practical engineering, it is necessary to carry out cyclic loading training in advance, namely: pre-loading and unloading 15 cycles, the mechanical properties of SMA wire can be stabilized.

Tab. 4 - Mechanical properties of austenite SMA wire at different loading / unloading cycles

Cycles	σ_a /MPa	σ_b /MPa	σ_c /MPa	σ_d /MPa	ΔW /MJ.m ⁻³	ζ_a /%
1	604.79	604.79	273.75	178.25	6.843	6.11
2	560.23	572.96	254.65	171.89	6.190	5.81
3	541.13	560.23	241.92	171.89	5.796	5.44
5	515.66	541.13	241.92	165.52	5.481	5.18
10	483.83	509.30	222.82	159.15	5.035	4.76
15	440.73	496.56	222.82	159.15	4.769	4.48
20	439.27	483.83	216.45	152.79	4.603	4.37
25	432.90	477.46	216.45	152.79	4.461	4.18
30	432.90	477.46	216.45	152.79	4.438	4.16



(a) Variation of stress-strain curves (b) Stresses of the characteristic points

Fig. 4 – Effect of loading / unloading cycles on the mechanical properties of austenite SMA wire

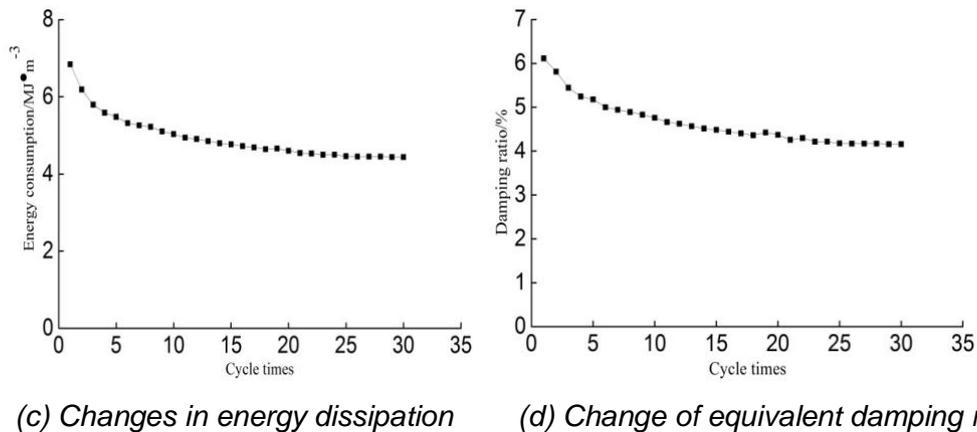


Fig. 4 – Effect of loading / unloading cycles on the mechanical properties of austenite SMA wire

Design of SMA suspension pendulum damping system

According to the results of performance test for SMA wire material, by the use of the superelastic properties of SMA wire, and in combination with the principle of the suspension pendulum damping, a new kind of SMA composite suspension pendulum damping system is designed and fabricated, as shown in Figure 5. The system is composed of a mass oscillator, a pendulum rod, a one-way hinge, a sliding block, a steering pulley and a trained SMA wire. Its basic structural design includes: the housing shell of the composite SMA pendulum-damping system (11); a one-way hinge (9) fixed at the top of its inside, connected with the upper end of the pendulum rod (1), perpendicular to the pendulum plane; where the plane, with its upper end inserted into the inlet hole, can freely go through the one-way hinge shaft; at both ends of the shaft, there are screw thread sleeves with the nuts to limit the shaft position; the middle part of the shaft is smooth enough to ensure that the pendulum swings freely; the lower end of the swinging rod is connected with the quality oscillator (2) at the centre of the screw hole, with symmetrical multiple mounting points arranged on the quality oscillator for the convenience of adjusting its qualities; the quality oscillator is provided with sliders (8), at the given distance of which are provided with the limit-setting devices (the baffles) (10); the SMA wire is linked with the sliders through the baffles so that pre tension force is provided by the baffles; post-stretching sliders are adjacent to the baffles; SMA wire goes through the bottom to the steering pulley (6) and then is linked with the cable (4) by means of the wire cable transfer (3); the cable goes through the upper part to the steering pulley (7), turns upwards the cable channel (12) throughout the housing and gets fixed with its outside structure.

Take one cycle as an example to illustrate the working principle and process of the SMA suspension pendulum damping system: When the earthquake is minor, the mass oscillator is not in contact with the sliding block, can freely swing, and the reverse inertia force is acted on the structure through the rigid outer wall. When the earthquake is major, the mass oscillator and the sliders move together. If the structure vibrates to the right, the mass oscillator will swing to the left and drive the right slider to slide along the horizontal chute. Meanwhile, the right SMA wire is pulled to produce a relative displacement and the left SMA wire is still in a static state. When the mass oscillator returns to the equilibrium position, the SMA wire recovers back to the initial pre-tensioned state. The right SMA wire has undergone a cycle of energy dissipation in the form of relatively full hysteresis curve, and realizes the energy dissipation damping control of the structure. At the same time, the inertia force of the mass vibrator is imposed on the structure through the steel cable, and the seismic response of the structure is restrained, and the seismic response of

the structure is reduced. Likewise, when the mass oscillator moves to the right, the principle is the same.

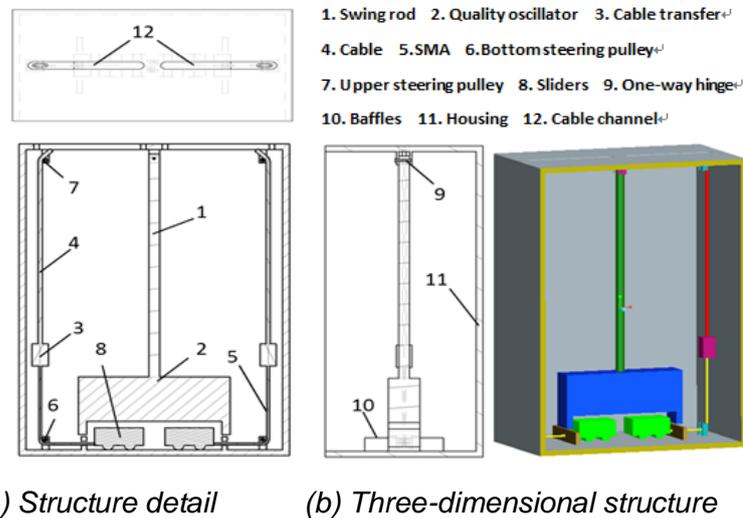


Fig. 5 – Schematic diagram of SMA composite suspension pendulum damping system

Experimental study on the performance of SMA suspension pendulum damping system

The experimental research on the performance of SMA suspension pendulum damping system are performed to primarily analyse the variation regularity for frequency, the phase relationship between the vibrator and the table top, equivalent damping force of this damping system with the mass of the vibrator and length of pendulum, as shown in Table 5. The experiment was completed on the WS-Z30-50 shaking table at Key Lab of Structure Engineering and Earthquake Resistance, Xi'an University of Architecture and Technology, as shown in Figure 6. In the experiment, the X direction displacement sensors and acceleration sensors are located in the mass oscillator. At the same time, in order to analyse the vibration phase relationship between the damping system and the table, the displacement and acceleration sensors are arranged at the table in the X direction.

Tab. 5- Test cases of SMA composite suspension pendulum system

Number	M (kg)	Length of pendulum (cm)	Wire diameter (mm)	Strain amplitude
1	15	35	0.5	6%
2	15	45	-	-
3	15	55	-	-
4	20	35	-	-
5	20	45	-	-
6	20	55	-	-
7	30	35	-	-
8	30	45	-	-
9	30	55	-	-
10	45	35	-	-
11	45	45	-	-
12	45	55	-	-

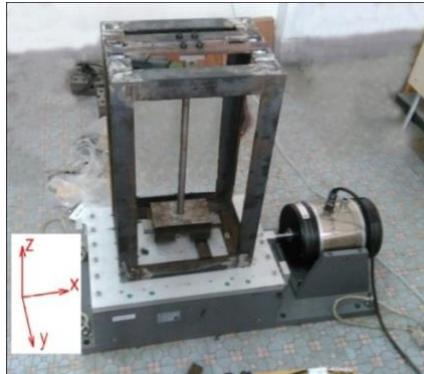


Fig. 6 – Performance test photo of SMA composite suspension pendulum damping system

Frequency analysis

The initial displacement method is used to get the displacement attenuation curve of the damping system, and the corresponding FFT frequency content analysis is carried out to obtain the vibration frequency of the SMA composite suspension pendulum system, as shown in Table 6.

From Table 6, it can be concluded that the natural vibration frequency of the designed damping system is near 0.8Hz. With the increase of the cycloid length, the natural frequency decreases gradually. In the meanwhile, with increasing the mass of the vibrator, the natural frequency gradually increases.

Tab. 6 - Natural frequencies of the damping system

Number	Natural frequency (Hz)	Number	Natural frequency (Hz)
1	0.77	7	0.86
2	0.75	8	0.85
3	0.74	9	0.85
4	0.78	10	0.88
5	0.76	11	0.86
6	0.75	12	0.87

Phase analysis

Whether the SMA composite suspension pendulum damping system can achieve the so-called "anti-resonance" of the controlled structure, depends on whether the mass vibrator can produce a "opposite" motion to the controlled structure [9,10]. If there is a lag or the same motion, the damping effect may disappear or even the dynamic response of the controlled structure may be amplified. Therefore, the SMA composite suspension pendulum damping system should be reasonably designed so that the relative displacement of the mass vibrator to the controlled structure could be totally opposite to the table velocity and thus the optimal damping effect can be realized [11-13].

Tab. 7 - Phase relationship of SMA composite suspension pendulum system

Number	Phase relation (°)	Number	Phase relation (°)
1	152	7	160
2	155	8	164
3	165	9	158
4	154	10	175
5	160	11	165
6	164	12	155

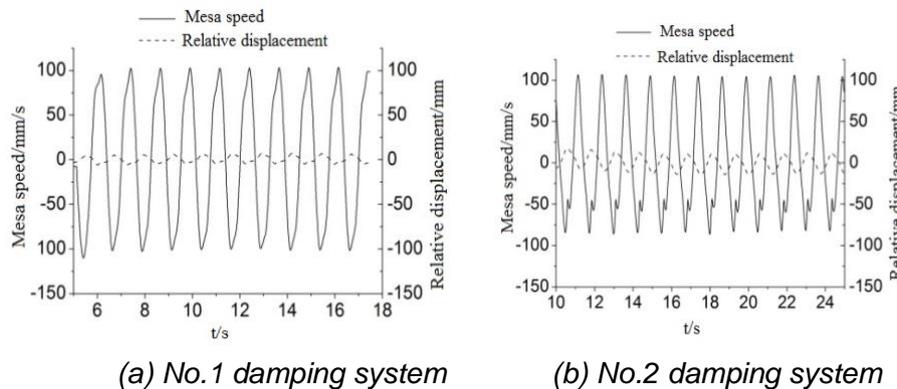


Fig. 7 – Phase relationship between the relative displacement of the mass oscillator and the velocity of the table

According to the above natural frequency of the damping system, the resonance test method under sine excitation was used to force the system to vibrate in the vicinity of the natural frequency (0.8Hz), so as to analyse the phase relationship of mass oscillator between the table top displacement and velocity direction. Figure 7 shows the time-history curves of the displacement and velocity of mass vibrator relative to the table for the 1 and 2 damping systems. Table 7 shows the phase relationship between the mass vibrator and the table of the damping systems 1 to 12.

Equivalent damping force of SMA composite suspension pendulum damping system

In describing the output performance of SMA composite suspension pendulum damping system, the following mechanical parameters are defined. The controlled structure is simplified to a single degree of freedom system. The dynamic equations of the controlled system (the original structure) and the vibration reduction system under the vibratory load are as follows [14].

$$M \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 = F(t) + F_e(t) \tag{1}$$

$$m \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = m \ddot{x}_1 \tag{2}$$

Where: $F_e(t)$ is the equivalent damping force supplemented to the original structure. From the interaction between the controlled structure and the damping system, the equivalent damping force can be calculated by Eq.(3).

$$F_e(t) = -m(\ddot{x}_1 + \ddot{x}_2) \tag{3}$$

For the above 12 kinds of dampers, the investigations on the varying laws of the equivalent damping force are conducted for SMA composite suspension pendulum system under the action of sine and real earthquake loads. When the sinusoidal load is applied, the displacement amplitudes

are 2mm, 4mm, 6mm, 8mm respectively; the loading frequency is 0.8Hz. When the real earthquake load is EL-Centro wave, the acceleration is 100gal, 200gal, 300gal, 400gal, 500gal respectively. The time history curves of the equivalent damping force for the 8 damping system are listed in Figure 9 under sinusoidal load (amplitude 6mm) and 400gal EL-Centro wave. Figure 10 is the amplitude change of the equivalent damping force 1~12 number of the damping system respectively under sine wave and EL-Centro wave.

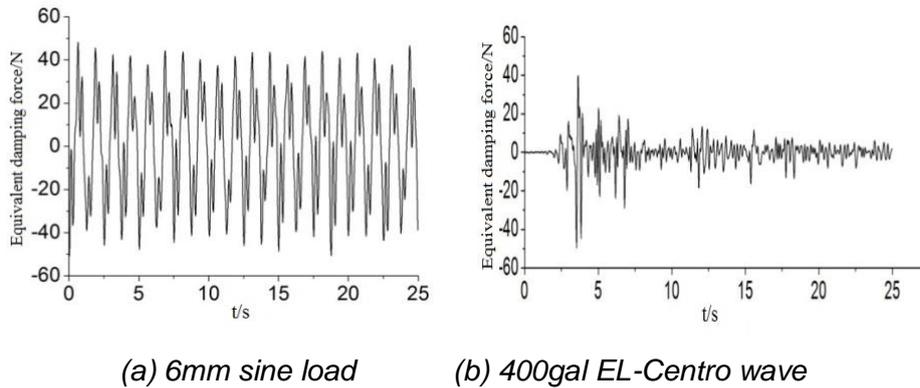


Fig. 9 – Time history of the equivalent damping force for No. 2 SMA composite suspension pendulum damping system

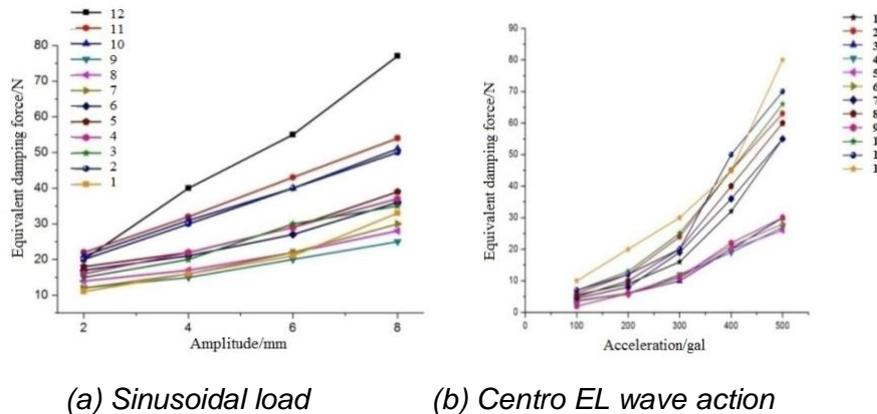


Fig. 10 – 12 Amplitude variation of the equivalent damping force of the SMA composite suspension pendulum systems

The above experimental results indicate that: the proposed SMA composite suspension pendulum damping system has a good output performance, and the equivalent damping force increases with the load amplitude. The mass of the vibrator yields a great impact on the equivalent damping force. Under the same load input, the equivalent damping force of the damping system is evidently improved with the increase of the oscillator mass, and the maximum increase is high up to 43%, but the length of the pendulum rod presents less influence on the equivalent damping force. At the meantime, the equivalent damping force of the damping system increases with the increase of earthquake intensity. The control effect of SMA composite suspension pendulum damping system is more remarkable for the structure under the strong vibratory loads.

CONCLUSIONS

A systematic study is conducted on variation regularities for such properties of SMA wires with the diameter of wires, strain amplitude and loading cycles as the stress-strain curve,

characteristic stresses, energy dissipation capacity, equivalent damping ratio. On the basis of the aforementioned regularities, choosing the appropriate and trained SMA wire and combined with the operating principles of TMD, a new kind of SMA suspension pendulum damping system, which is easy to disassemble, was designed and fabricated. The experimental researches were carried out on the SMA suspension pendulum damping system.

(1) With increasing the diameter of SMA wire, the mechanical properties of SMA wire are decreased in different amplitudes. With the increase of the number of loading cycles, the values of the characteristic points decrease in different degrees, but after 15 loading cycles, those values tend to be stable. Therefore, based on the above-mentioned law, the 15-cycle trained SMA wire with a diameter of 0.5mm and strain amplitude of 6% is selected for the exploitation of SMA-SPDS system.

(2) The phase analysis of the SMA composite suspension pendulum system was carried out by using the resonance test method under sinusoidal excitation. The results show that the phase relationship between the relative displacement of the mass oscillator and the controlled structure and the velocity of the table always ranges from 150° to 180° . So the real time reliable and effective damping force can be offered by this damping system for the controlled structure.

(3) The 12 kinds of SMA composite suspension pendulum damping system with different pendulum mass and different pendulum length are excited by sine wave with different displacement amplitudes and EL-Centro waves with different intensities. The results indicate that the equivalent damping force of the damping system is obviously improved with the increase of the oscillator mass, the maximum increase high up to 43%, but the effect of the length of the pendulum rod on the equivalent damping force of the damping system is slight. In the meanwhile, the equivalent damping force of the damping system increases with the increase of earthquake intensity. So the control effect of SMA composite suspension damping system is more considerable for the structure subjected to the strong vibration load.

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