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A METHOD FOR DETERMINING RAYLEIGH DAMPING PARAMETERS OF COMPLEX FIELD

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ABSTRACT

Rayleigh damping model, which is still adopted by many general finite element programs and widely used in analysis of engineering, leads to the inconformity of the calculating modal damping ratio with the actual modal damping ratios. The appropriate Rayleigh damping matrix is significant for accurate dynamic response analysis of complicated field. This paper establishes a dual parameter optimization theory for calculation of Rayleigh damping coefficients. The functional relation between the relative error of dynamic response and Rayleigh damping coefficients is established based on CQC method. By taking the square sum of the errors of peak value of displacement and the error of peak value of acceleration at the multiple points (DOF) of the surface of the complex site as the control objective, the equations for solving Rayleigh damping coefficients are obtained based on the principle of minimizing the control objective. Then, as an example, the seismic response of a valley under the excitation of 28 representative seismic waves which are randomly selected is calculated and the error due to Rayleigh damping model is analysed. The numerical result verifies the accuracy and applicability of the proposed method.

KEYWORDS

Rayleigh damping, Damping parameters, Complicated field, Seismic response analysis

INTRODUCTION

Direct time integration is an important analysis method in seismic response analysis, especially when nonlinearity is involved. A number of procedures are available for the modelling of damping in time-domain analysis. Viscous damping is routinely assumed, and Rayleigh damping is a popular choice (the main reasons being that Rayleigh damping preserves the undamped natural modes of the system as discussed below and, sometimes, that most finite element codes offer few other models to choose from). It is generally acknowledged that there is little physical evidence to support Rayleigh damping. Many real systems encountered in civil engineering practice display hysteretic damping which is largely independent of frequency[1]. Modal damping, which is constant for all frequencies, is the damping typically specified in seismic analysis Codes and Standards[18,19]. On a more fundamental level, it can be shown that the mass-proportional damping matrix does not remain invariant under a Galilean transformation as it must do to comply with the classical principle of relativity[2]. Despite these limitations, Rayleigh damping can and has been used as a heuristic, as opposed to strictly physical, attenuation mechanism.

When Rayleigh damping is specified, the damping matrix $[C]$ is linearly dependent on the mass and stiffness matrices, $[M]$ and $[K]$, such that:

$$[C]=\alpha[M]+\beta[K] \tag{1}$$

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where α and β are real scalars, called Rayleigh damping coefficients. Rayleigh damping belongs to the group of classical damping models: this implies that the damping matrix satisfies an orthogonality condition:

$$\phi_i^T [C] \phi_j = \begin{cases} 2\zeta_i \omega_i, i = j \\ 0, i \neq j \end{cases} \tag{2}$$

where ω_i and ϕ_j are the undamped natural frequency and modal shape of mode i and ζ_i is the modal damping ratio of mode i, which for Rayleigh damping is given by the familiar formula:

$$\zeta_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \tag{3}$$

Rayleigh damping coefficients, α and β, can be determined by selecting the two reference modal frequencies (ω_j and ω_k) of the dynamic system and the corresponding modal damping ratios (ζ_j and ζ_k). Thus, except the two selected reference modes, the modal damping ratios of the other modes calculated by the Rayleigh damping matrix are inconsistent with the actual modal damping ratios, and the contribution of these modes to the dynamic response of system is either overestimated or underestimated, which deviates the calculation result of dynamic response of system.

Rayleigh damping matrix is widely used in engineering calculation and research. However, there is lack of deep understanding of the influence of Rayleigh damping matrix on the numerical calculation of dynamic response of system, mainly because the fundamental frequencies of a large number of engineering structures in the past are higher or close to the frequency of the main component of the external dynamic excitation and the low-order mode of the system plays a major role in the total dynamic response of the system. Therefore, two low-order modes of the system are usually selected to establish the Rayleigh damping matrix, as at this time, the calculated damping ratio of low-order mode is equal to the actual damping ratio, and thus the calculation error caused by Rayleigh damping matrix is relatively slight. The study done by Hashash et al.[3] and Lou et al.[4] show that when the fundamental frequency of the system is much lower than the frequency of the main component of the external dynamic excitation, if only the frequencies and damping ratios corresponding to the two low-order modes of the system are selected to calculate α and β, the dynamic response of system will be underestimated. Considering that, some researchers [5, 6] have proposed methods to reconstruct the damping matrix on the base of extensional Rayleigh damping matrix. However, this modification led to the abandonment of the proportional property of classical damping. What demands urgent solution is that Rayleigh damping matrix is still adopted by many general finite element programs[7] and widely used in analysis of engineering[8]. Meanwhile, little specific guidelines or evaluation criteria are presently available to guide engineers or analysts for such an important selection. Thus, the selection of Rayleigh damping coefficients, which is worthy of a deeper study, but not the reconstruction of Rayleigh damping matrix, is the purpose of this paper.

In order to improve the calculation error due to Rayleigh damping matrix, some scholars[1, 7, 9, 10] choose the specific frequencies (and their corresponding damping ratios) of the system or seismic wave to calculate α and β. These specific frequencies include the fundamental frequency of the dynamic system, high-order modal frequency which has a significant contribution to the dynamic response of system, the frequency corresponding to the peak or the barycenter of the response spectrum (or Fourier spectrum) of the external dynamic excitation, and the characteristic frequency of the site, etc.

With a purpose to avoiding arbitrarily and empirically choosing the reference mode in establishing Rayleigh damping matrix, the standard least-squares method, a widely used mathematical method, is used to determine Rayleigh damping coefficients[11]. Yang et al. [12]

proposed that $\gamma_i^2 M_i^* S_{vi}^2$ (where γ_i is the participation coefficient of mode i, M_i^* is the generalized mass of mode i, and S_{vi} is the pseudo-velocity of the single-degree-of-freedom (SDOF) system corresponding to mode i) can serve as weighting coefficient. Pan et al.[13] suggested using $\gamma_i^2 \phi_{ki}^2 S_{ui}'^2$ (where ϕ_{ki} is the k^{th} element of the vector of mode i, S_{ui}' is the derivative of the displacement of the SDOF system corresponding to mode i to modal damping ratio) as the weighting factor. And the author of this paper[14] also proposed adopting $|H|_i^2 |H'|_i^2 |F|_i^2$ (where H_i is the amplitude of frequency response function corresponding to mode i, H_i' is the amplitude of the derivative of frequency response function corresponding to mode i to modal damping ratio, and F_i is the amplitude of Fourier spectrum of external dynamic excitation acceleration corresponding to mode i) as the weighting coefficient. These are dual parameter optimization methods. Dong et al.[15] and the author of this paper[16] introduced respectively the first-order mode of the system to propose a single-parameter optimization method. In addition to the weighted least squares method, Spears et al.[17] proposed making $\sum_{i=1}^{n} M_{ei} A_i$ (where M_{ei} is the effective mass of mode i, A_i is the maximum acceleration response of the SDOF system corresponding to mode i) equal to zero (or slightly greater than zero) as the objective, and solving α and β by iterative method.

Major engineering structures generally require seismic response analysis of the complex site. Meanwhile, the fundamental frequency of site with deep deposit is often low. For example, the thickness of the soil layer in Shanghai (China) is about 300 m and its fundamental frequency is about 0.5 Hz, much lower than the frequency of the main component of most of seismic wave from bedrock. To accurately calculate the seismic response of such site, Rayleigh damping matrix must be properly established in the first place. However, there are some drawbacks, if the aforementioned methods were adopted in seismic analysis of the complex site, as follows:

① Because the response at only one degree of freedom (DOF) can serve as the control objective, these methods are only applicable to horizontally stratified soil site which can be usually simplified as soil column. When they are applied to complex site, the contribution ratio of each mode to DOF is inconsistent, which is related to ϕ_{ki}. When the error of dynamic response at one DOF is taken as the control objective, there is the possibility that the error of dynamic response at DOF is not slight

② These methods only select one kind of dynamic response as the control objective, while the contribution ratio of each mode to different dynamic responses is varied, which is related to S_{ri} (where S_{ri} is the dynamic response of the SDOF system corresponding to mode i). In these methods, only the error of one kind of dynamic response is selected as the control objective, which may lead to non-negligible errors of the other kinds of dynamic response

③ The calculation error is not small enough for engineering practice.

Therefore, on the basis of the previous study, this paper establishes a dual parameter optimization theory for calculation of Rayleigh damping coefficients (α and β). By taking the square sum of the error of peak value of displacement and the error of peak value of acceleration at the multiple points (DOF) of the surface of the complex site as the control objective, the equations for solving Rayleigh damping coefficients are obtained based on the principle of minimizing the control objective. Then, as an example, the seismic response of a valley under the excitation of 28 representative seismic waves which are randomly selected is calculated and the error due to Rayleigh damping model is analysed. The numerical result verifies the accuracy and applicability of the proposed method.

DUAL PARAMETER OPTIMIZATION THEORY FOR RAYLEIGH DAMPING COEFFICIENTS

Under the input of consistent ground motion, the vibration equation of multi-degree-of-freedom (MDOF) system can be expressed as:

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=-[M]\{1\}\ddot{u}_g(t) \tag{4}$$

where: $\{1\}$ is a column vector of all elements which are 1, $\ddot{u}_g(t)$ is the acceleration time history of the ground motion, and $\{\ddot{u}(t)\}$, $\{\dot{u}(t)\}$ and $\{u(t)\}$ are the column vectors of relative acceleration, relative velocity and relative displacement time history.

The contribution of mode i ($|r_{ki}|_{\max}$) to the maximum dynamic response of the k^{th} DOF ($|r_k|_{\max}$) can be expressed as:

$$|r_{ki}|_{\max}=\gamma_i\phi_{ki}S_{ri}(\zeta_i) \tag{5}$$

where: γ_i is the participation coefficient of mode i, $S_{ri}(\zeta_i)$ is the maximum dynamic response of the SDOF system corresponding to mode i, and here the dynamic response can be displacement, velocity, acceleration or others.

Based on complete quadratic combination (CQC) method, the maximum dynamic response of the k^{th} DOF ($|r_k|_{\max}$) can be expressed as:

$$|r_k|_{\max}=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ri}(\zeta_i)S_{rj}(\zeta_j)\right)} \tag{6}$$

where: ρ_{ij} is the coupling coefficient of mode i and mode j and n is the total number of degree of MDOF system.

When $S_{ri}(\zeta_i)$ is expanded by the first-order Taylor series at the accurate damping ratio of mode i, it can be found as follows:

$$S_{ri}(\zeta_i)\approx S_{ri}(\zeta_i^*)+S_{ri}'(\zeta_i^*)(\zeta_i-\zeta_i^*) \tag{7}$$

where: $S_{ri}'(\zeta_i)=\dfrac{\partial S_{ri}(\zeta_i)}{\partial\zeta_i}$ is the partial derivative of $S_{ri}(\zeta_i)$ to the damping ratio of mode i, and the item with "*" corresponds to the accurate damping ratio. Thus, the absolute error of $|r_{ki}|_{\max}$ can be expressed as:

$$\begin{aligned}E^{|r_{ki}|_{\max}}&=\gamma_i\phi_{ki}\left[S_{ri}(\zeta_i^*)+S_{ri}'(\zeta_i^*)(\zeta_i-\zeta_i^*)\right]-\gamma_i\phi_{ki}S_{ri}(\zeta_i^*)\\&=\gamma_i\phi_{ki}S_{ri}'(\zeta_i^*)(\zeta_i-\zeta_i^*)\end{aligned} \tag{8}$$

Based on CQC method, the absolute error of $|r_k|_{\max}$ can be expressed as:

$$E^{|r_k|_{\max}}=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ri}'(\zeta_i^*)S_{rj}'(\zeta_j^*)(\zeta_i-\zeta_i^*)(\zeta_j-\zeta_j^*)} \tag{9}$$

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Thus, the relative error of $|r_k|_{\max}$ can be expressed as:

$$e^{|r_k|_{\max}} = \frac{E^{|r_k|_{\max}}}{|r_k|_{\max}} = \frac{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ri}'\left(\zeta_i^*\right)S_{rj}'\left(\zeta_j^*\right)\left(\zeta_i-\zeta_i^*\right)\left(\zeta_j-\zeta_j^*\right)}}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ri}\left(\zeta_i^*\right)S_{rj}\left(\zeta_j^*\right)\right)}} \tag{10}$$

When the displacement (u) or the acceleration (a) is selected as the dynamic response, $S_{ri}\left(\zeta_i\right)$ can be expressed as:

$$S_{ui}\left(\zeta_i\right) = \left|-\frac{1}{\omega_{iD}}\int_0^t \ddot{u}_g\left(\tau\right)\exp\left[-\zeta_i\omega_i\left(t-\tau\right)\right]\sin\left[\omega_{iD}\left(t-\tau\right)\right]d\tau\right|_{\max} \tag{11}$$

$$S_{ai}\left(\zeta_i\right) = \left|\frac{\omega_i^2}{\omega_{iD}}\int_0^t \ddot{u}_g\left(\tau\right)\exp\left[-\zeta_i\omega_i\left(t-\tau\right)\right]\sin\left[\omega_{iD}\left(t-\tau\right)+2\theta_i\right]d\tau\right|_{\max} \tag{12}$$

where: $\omega_{iD} = \omega_i\sqrt{1-\zeta_i^2}$ is the damped circular frequency of mode i , and $\theta_i = \arctan\left(\frac{\zeta_i}{\sqrt{1-\zeta_i^2}}\right)$ is the phase angle of mode i .

By substituting the Equation (3), Equation (11) and Equation (12) into Equation (10), the relative error of the maximum displacement ($e^{|u_k|_{\max}}$) and that of maximum acceleration ($e^{|a_k|_{\max}}$) can be obtained, which respectively are:

$$e^{|u_k|_{\max}} = \frac{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}'\left(\zeta_i^*\right)S_{uj}'\left(\zeta_j^*\right)\left(\frac{\alpha}{2\omega_i}+\frac{\beta\omega_i}{2}-\zeta_i^*\right)\left(\frac{\alpha}{2\omega_j}+\frac{\beta\omega_j}{2}-\zeta_j^*\right)}}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}\left(\zeta_i^*\right)S_{uj}\left(\zeta_j^*\right)\right)}} \tag{13}$$

$$e^{|a_k|_{\max}} = \frac{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}'\left(\zeta_i^*\right)S_{aj}'\left(\zeta_j^*\right)\left(\frac{\alpha}{2\omega_i}+\frac{\beta\omega_i}{2}-\zeta_i^*\right)\left(\frac{\alpha}{2\omega_j}+\frac{\beta\omega_j}{2}-\zeta_j^*\right)}}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}\left(\zeta_i^*\right)S_{aj}\left(\zeta_j^*\right)\right)}} \tag{14}$$

To control the errors of displacement and acceleration at multiple points (DOF) of site simultaneously, let:

$$e^2 = \sum_{k=1}^{m}\left(e^{|u_k|_{\max}}\right)^2 + \sum_{k=1}^{m}\left(e^{|a_k|_{\max}}\right)^2 \tag{15}$$

where: m is the total number of all points (DOF) of interest in the analysis.

To get the minimum value of e^2 , the partial derivative of e^2 with respect to α and β set to be zero, and the equations for solving α and β can be obtained:

$$\begin{cases} \frac{\partial e^2}{\partial\alpha} = \alpha\sum_{k=1}^{m}\left(A_{11}^k + B_{11}^k\right) + \beta\sum_{k=1}^{m}\left(A_{12}^k + B_{12}^k\right) + \sum_{k=1}^{m}\left(A_{13}^k + B_{13}^k\right) = 0 \\ \frac{\partial e^2}{\partial\beta} = \alpha\sum_{k=1}^{m}\left(A_{21}^k + B_{21}^k\right) + \beta\sum_{k=1}^{m}\left(A_{22}^k + B_{22}^k\right) + \sum_{k=1}^{m}\left(A_{23}^k + B_{23}^k\right) = 0 \end{cases} \tag{16}$$

where:

$$\begin{cases} A_{11}^k = \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}'\left(\zeta_i^*\right)S_{uj}'\left(\zeta_j^*\right)\left(\dfrac{1}{2\omega_i}\right)\left(\dfrac{1}{2\omega_j}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}\left(\zeta_i^*\right)S_{uj}\left(\zeta_j^*\right)\right)} \\ A_{12}^k = A_{21}^k = \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}'\left(\zeta_i^*\right)S_{uj}'\left(\zeta_j^*\right)\left(\dfrac{1}{2\omega_i}\right)\left(\dfrac{\omega_j}{2}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}\left(\zeta_i^*\right)S_{uj}\left(\zeta_j^*\right)\right)} \\ A_{22}^k = \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}'\left(\zeta_i^*\right)S_{uj}'\left(\zeta_j^*\right)\left(\dfrac{\omega_i}{2}\right)\left(\dfrac{\omega_j}{2}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}\left(\zeta_i^*\right)S_{uj}\left(\zeta_j^*\right)\right)} \\ A_{13}^k = -\dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}'\left(\zeta_i^*\right)S_{uj}'\left(\zeta_j^*\right)\zeta_j^*\left(\dfrac{1}{2\omega_i}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}\left(\zeta_i^*\right)S_{uj}\left(\zeta_j^*\right)\right)} \\ A_{23}^k = -\dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}'\left(\zeta_i^*\right)S_{uj}'\left(\zeta_j^*\right)\zeta_i^*\left(\dfrac{\omega_j}{2}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ui}\left(\zeta_i^*\right)S_{uj}\left(\zeta_j^*\right)\right)} \end{cases} \tag{17}$$

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$$\begin{cases} B_{11}^k = \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}'\left(\zeta_i^*\right)S_{aj}'\left(\zeta_j^*\right)\left(\dfrac{1}{2\omega_i}\right)\left(\dfrac{1}{2\omega_j}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}\left(\zeta_i^*\right)S_{aj}\left(\zeta_j^*\right)\right)} \\ B_{12}^k = B_{21}^k = \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}'\left(\zeta_i^*\right)S_{aj}'\left(\zeta_j^*\right)\left(\dfrac{1}{2\omega_i}\right)\left(\dfrac{\omega_j}{2}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}\left(\zeta_i^*\right)S_{aj}\left(\zeta_j^*\right)\right)} \\ B_{22}^k = \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}'\left(\zeta_i^*\right)S_{aj}'\left(\zeta_j^*\right)\left(\dfrac{\omega_i}{2}\right)\left(\dfrac{\omega_j}{2}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}\left(\zeta_i^*\right)S_{aj}\left(\zeta_j^*\right)\right)} \\ B_{13}^k = -\dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}'\left(\zeta_i^*\right)S_{aj}'\left(\zeta_j^*\right)\zeta_j^*\left(\dfrac{1}{2\omega_i}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}\left(\zeta_i^*\right)S_{aj}\left(\zeta_j^*\right)\right)} \\ B_{23}^k = -\dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}'\left(\zeta_i^*\right)S_{aj}'\left(\zeta_j^*\right)\zeta_i^*\left(\dfrac{\omega_j}{2}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\rho_{ij}\gamma_i\gamma_j\phi_{ki}\phi_{kj}S_{ai}\left(\zeta_i^*\right)S_{aj}\left(\zeta_j^*\right)\right)} \end{cases} \tag{18}$$

MODEL ANALYSIS

To investigate the validity and accuracy of the proposed method, the finite element model of a valley was established and the seismic analysis was done.

The valley and its finite element model

The span of the valley is 500m. The slope of the left bank of the valley is 8.3%, and the slope of the right bank of the valley is 12.0%. The specific size of the valley is detailed in 0. 0 shows the parameters of each soil layer of the site. The fundamental frequency of the site is 0.76Hz.

A commercial software product for finite element analysis, ANSYS, is used to establish the two-dimensional finite element model of the valley and PLANE42 element is used to model soil body. This paper studies the linear problem without any consideration on the nonlinear characteristics of the soil. Fixed constrains are applied at the bottom of the soil to simulate rigid bedrock where the seismic waves come. In view of the radiation damping of semi-infinite space, the horizontal scope of soil is set large enough, extending $5H$ long from valley scope, being $H = 126m$ the total thickness of soil layers. And in view of the transmission of energy in soil, the finite element mesh is set small enough, limited to $1/8\lambda_{\min}$, being $\lambda_{\min}=c_s/f_{\max}$ the minimum premeditated wavelength. Here $f_{\max}=25Hz$ is the maximum premeditated frequency of seismic wave and c_s is the shear wave velocity of soil layer (0).

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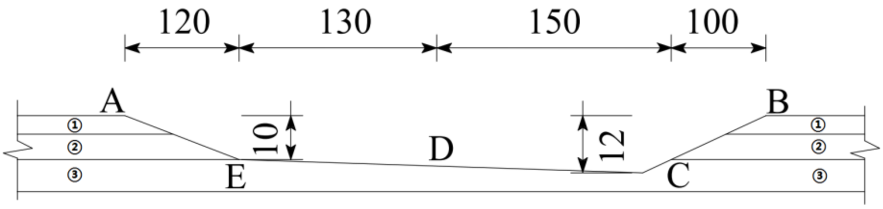


Fig - Cross-section of the valley (unit: m)

Tab. 1 - Parameters of soil layers

No.	Soil layer	Depth of soil layer (m)	Shear wave velocity (m/s)	Mass density (kg/m³)
1	Mud	3.5	105	1900
2	Muddy-silty clay	10.0	127	1720
3	Silty clay	17.0	147	1850
4	Sandy silt	20.5	173	2000
5	Silty clay	25.3	204	1830
6	Clay	29.2	244	1850
7	Silt	33.0	265	1880
8	Silt	49.0	305	1880
9	Silty clay	73.5	350	2020
10	Fine sand	91.0	394	1920
11	Silty clay	108.0	412	2020
12	Medium sand	126.0	440	1980
13	Rock	∞	-	-

Seismic wave

The seismic inputs include 25 ground motion records from bedrock with different epicentral distances and 3 artificial seismic waves with different exceedance probabilities. Because this paper studies linear problem, the peak values of all seismic waves are set to be 1 for comparative analysis. 0 shows the information of the seismic waves. 0 shows the acceleration time history and response spectrum of seismic wave (given space limitations, this paper only shows TMZ-000 seismic wave).

Tab. 2 - Earthquake ground motions

No.	Code name	Earthquake	Time
1	R1-3%	Artificial (exceedance probability 3%)	-
2	R1-10%	Artificial (exceedance probability 10%)	-
3	R1-63%	Artificial (exceedance probability 63%)	-
4	CPM-030	Northern California	1975/6/7
5	GGP-100	San Francisco	1957/3/22
6	GRN-180	Landers	1992/6/28
7	KAU-NS	Chi-Chi	1999/9/20
8	LAM-EW	Duzce	1999/11/12
9	LIT-180	Northridge	1994/1/17
10	LUA-CUT	Wenchuan	2008/5/12
11	MCH-000	Loma Prieta	1989/10/18
12	MSK-EW	Kocaeli	1999/8/17
13	TTN-NS	Chi-Chi, Taiwan	1999/9/20
14	TVY-045	Anza (Horse Cany), USA	1980/2/25
15	WON-075	Whittier Narrows	1987/10/1
16	C08-050	Parkfield, USA	1966/6/28
17	CPE-045	Victoria, Mexico	1980/6/9
18	DVD-246	Livermore, USA	1980/1/24
19	EIL-EW	Aqaba, Jordan	1995/11/22
20	ELC-NS	Imperial Valley, USA	1940/5/18
21	FER-T1	Tabas, Iran	1978/9/16
22	MLS-270	Mammoth Lakes, USA	1980/5/25
23	ORR-090	Northridge, USA	1994/1/17
24	S3-270	Nahanni, Canada	1985/12/23
25	SHP-010	Victoria, Mexico	1980/6/9
26	STG-000	Loma Prieta, USA	1989/10/18
27	TAZ-090	Kobe, Japan	1995/1/16
28	TMZ-000	Friuli, Italy	1976/5/6

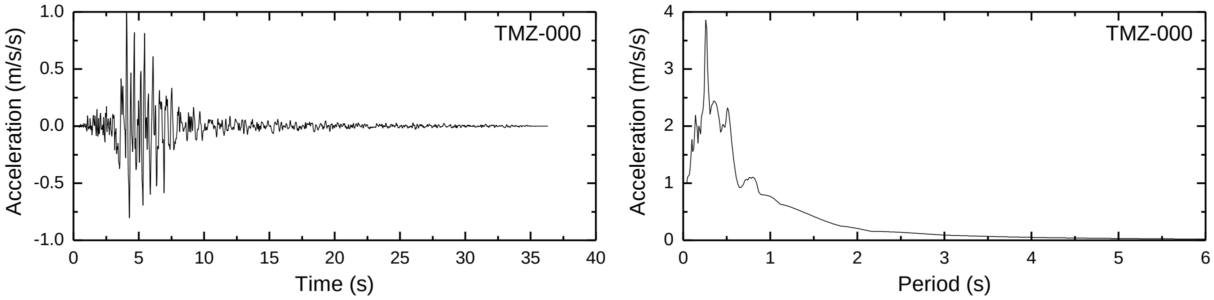


Fig 2 - Acceleration time histories and its spectrum

Calculation method for Rayleigh damping coefficients

The proposed method in this paper is adopted to calculate the Rayleigh damping coefficients (termed as M4) and the controlled points (DOF) choose A, B, C, D, E, a total of 5 points (0). In addition, the Rayleigh damping coefficients are calculated using the new methods recommended by Lou[9], Pan[13], and the author in a previous work[16] for comparison. These methods are termed as M1, M2, and M3, respectively.

Numerical result

To measure the calculation error caused by Rayleigh damping model, the accurate solution of seismic response analysis is obtained by using the modal superposition method and the modal damping ratio of each mode is assumed to be 5% The modal damping ratio of each mode is assumed to be equal, which is widely used in seismic response analysis[9]. The rationality of this assumption is not the focus in this paper, but the proposed method in this paper is not limited to this case, which is applicable when the modal damping ratio of each mode is not equal.

The calculated relative error caused by Rayleigh damping is inspected here by using the error of dynamic response at points A, B, C, D, E on the surface of valley. The calculated relative error of a point is defined as:

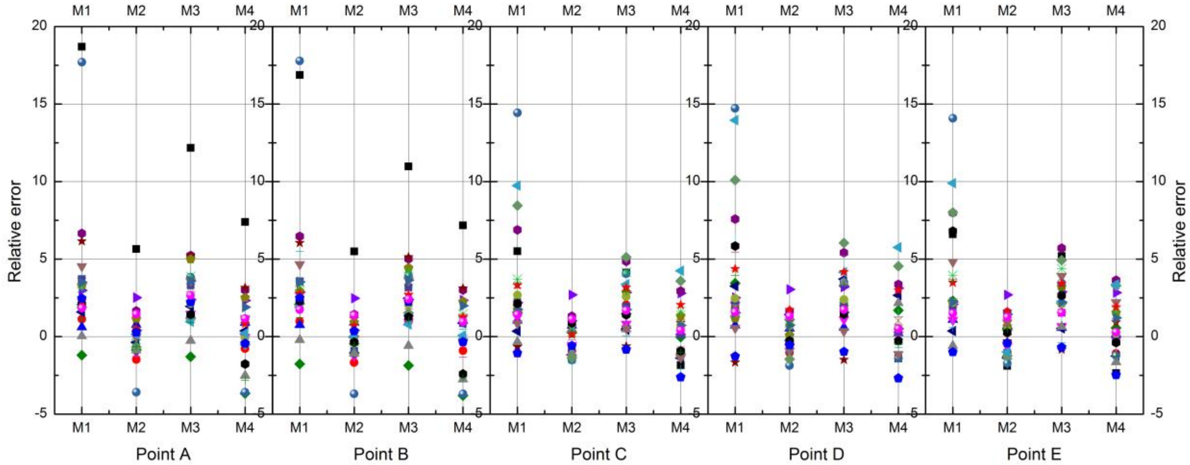
$$e = \frac{\left|r^R\right|_{\max} - \left|r^*\right|_{\max}}{\left|r^*\right|_{\max}} \times 100\% \tag{19}$$

where, $\left|r^*\right|_{\max}$ is the maximum dynamic response (displacement, velocity or acceleration) corresponding to the accurate damping, which is calculated by using modal superposition method, and $\left|r^R\right|_{\max}$ is the maximum dynamic response corresponding to the Rayleigh damping when the Rayleigh damping coefficients is calculated by using the methods advocated Lou[9], Pan[13], and the author[16] and this paper.

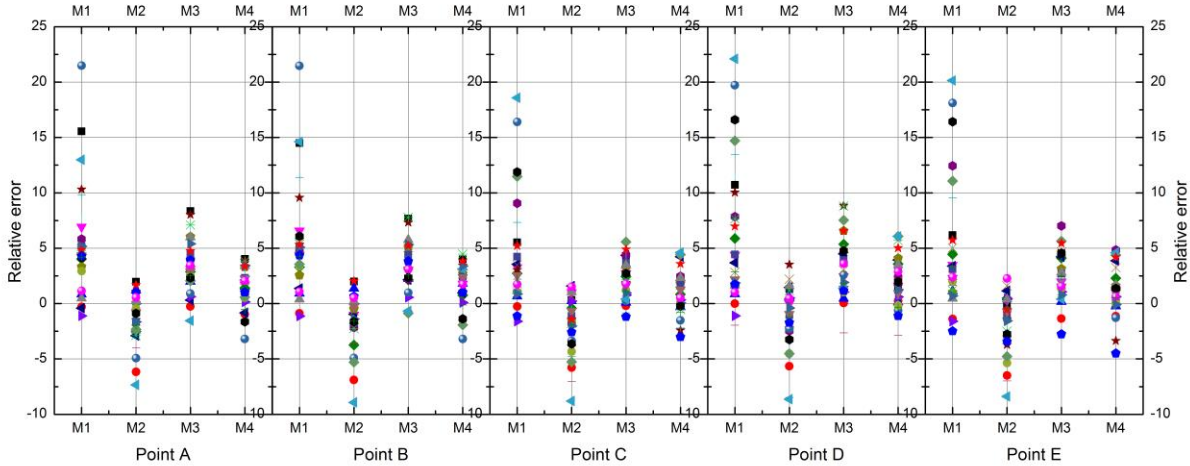
0 shows the two reference frequencies for calculation of Rayleigh damping coefficients, 0 shows the relative errors of peak values of displacements, velocities and accelerations at points A, B, C, D, E which are caused by Rayleigh damping. 0 shows the mean and standard deviation of the relative error (e).

Tab. 3 - Two reference frequencies for calculation of Rayleigh damping coefficients (unit: Hz)

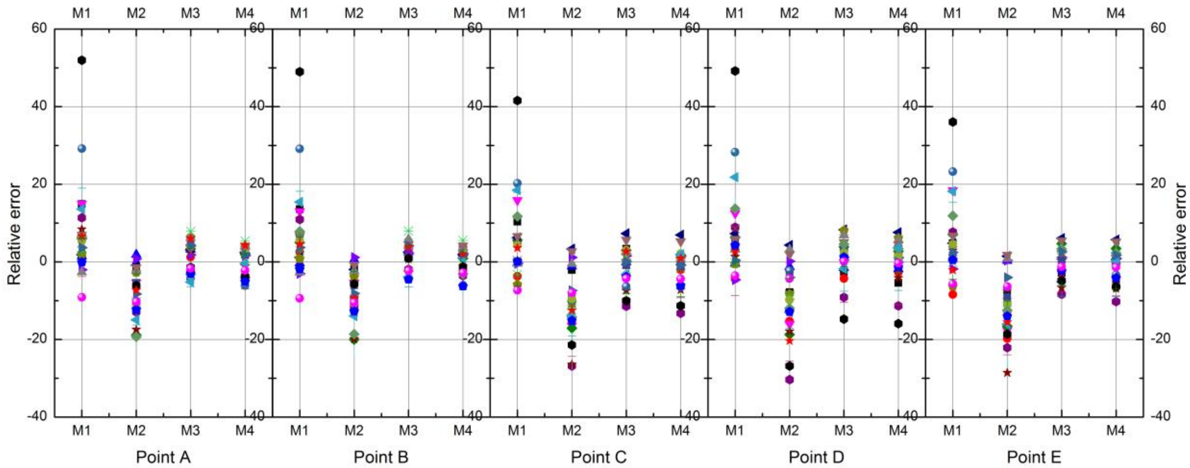
Code name	M1		M2		M3		M4	
	f_j	f_k	f_j	f_k	f_j	f_k	f_j	f_k
R1-3%	0.76	5.56	0.84	2.58	0.76	4.86	0.90	4.43
R1-10%	0.76	4.17	0.86	2.33	0.76	5.11	0.90	4.88
R1-63%	0.76	4.76	0.83	2.50	0.76	4.48	0.88	4.37
CPM-030	0.76	7.14	1.92	5.53	0.76	6.09	2.09	6.04
GGP-100	0.76	4.55	0.86	3.34	0.76	4.56	0.95	4.77
GRN-180	0.76	1.22	0.78	1.76	0.76	2.63	0.79	2.73
KAU-NS	0.76	10.00	0.81	1.79	0.76	3.97	0.85	4.27
LAM-EW	0.76	5.56	0.79	2.54	0.76	5.09	0.83	4.95
LIT-180	0.76	3.33	0.95	2.46	0.76	3.86	0.95	4.22
LUA-CUT	0.50	0.76	0.78	1.58	0.76	1.98	0.79	2.01
MCH-000	0.76	6.25	0.81	2.66	0.76	4.36	0.83	4.19
MSK-EW	0.76	5.88	0.91	2.40	0.76	4.89	1.23	4.97
TTN-NS	0.76	2.94	0.82	2.19	0.76	3.99	0.95	3.92
TVY-045	0.76	11.11	1.99	4.33	0.76	5.14	2.39	5.04
WON-075	0.76	10.00	0.83	3.39	0.76	4.86	0.90	4.99
C08-050	0.76	6.25	0.84	2.12	0.76	4.87	0.88	4.77
CPE-045	0.76	4.35	0.86	1.81	0.76	3.85	0.86	3.85
DVD-246	0.76	4.76	0.88	2.42	0.76	4.44	0.93	4.48
EIL-EW	0.76	2.22	0.81	2.30	0.76	3.78	0.86	3.58
ELC-NS	0.76	4.00	0.87	1.90	0.76	3.11	0.89	3.53
FER-T1	0.76	7.69	0.79	2.16	0.76	4.64	0.83	4.53
MLS-270	0.76	7.14	0.78	2.42	0.76	3.38	0.81	4.00
ORR-090	0.76	3.85	0.80	2.20	0.76	3.93	0.88	3.66
S3-270	0.76	16.67	0.79	2.74	0.76	4.84	0.91	4.86
SHP-010	0.76	4.55	0.78	2.13	0.76	4.37	0.81	4.06
STG-000	0.76	6.25	0.84	2.56	0.76	5.12	0.92	4.87
TAZ-090	0.76	2.13	0.79	1.99	0.76	3.30	0.85	3.38
TMZ-000	0.76	3.85	0.91	2.19	0.76	3.20	1.15	3.47



a) Relative error of peak value of displacement



b) Relative error of peak value of velocity



c) Relative error of peak value of acceleration

Fig. 3 - Relative errors of dynamic response (%)

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Tab 4. - Statistical data of relative errors (%)

Method	Displacement		Velocity		Acceleration	
	Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Standard deviation
M1	3.5	5.2	5.3	7.6	6.6	12.8
M2	0.1	1.3	-1.6	3.1	-10.6	13.3
M3	2.4	3.1	3.0	3.8	0.1	4.7
M4	0.4	2.0	1.4	2.5	-0.8	4.7

Analysis and discussion

When the Rayleigh damping coefficients are calculated using the method proposed by Lou[9] (M1), the relative error of the peak value of displacement is $-1.8\% \sim 18.7\%$, that of velocity is $-2.5\% \sim 22.1\%$, that of acceleration response is $-9.4\% \sim 52.0\%$, and the error of acceleration is the largest followed by the error of velocity. The displacement has the minimum error because the contribution ratio of the first mode of the system to the displacement is greater than to the velocity and acceleration. This method guarantees the accuracy of Rayleigh damping ratio of first mode when generating the Rayleigh damping matrix. The error of this method is relatively large and basically positive. Applying this method to engineering calculation will certainly make the calculation result too conservative. Before a more precise method is found, it is a suitable method for engineering practice.

When the Rayleigh damping coefficients are calculated using the method proposed by Pan[13] (M2), the relative error of the peak value of displacement is $-3.7\% \sim 5.7\%$, that of velocity is $-8.9\% \sim 3.5\%$, and that of acceleration response is $-30.4\% \sim 4.3\%$. Compared with the velocity and acceleration, the displacement has the slightest error, because this method takes the minimum error of the peak value of displacement as the control objective when establishing Rayleigh damping matrix. The errors of displacement and velocity are within $\pm 10\%$. Provided that the engineering calculation only concerns the displacement and velocity of the site, then this method can be applied. However, since the error of acceleration is large and negative almost in every case, adopting this method in engineering practice will make the calculation results relatively more dangerous.

When the Rayleigh damping coefficients are calculated using the method proposed by the author in a previous work[16] (M3), the relative error of the peak value of displacement is $-1.9\% \sim 12.2\%$, that of velocity is $-2.8\% \sim 9.1\%$, and that of acceleration response is $-14.8\% \sim 8.3\%$. In the case that the displacement, the velocity and the acceleration of the site are all concerned in the engineering calculation, compared with the methods recommended by Lou[9] (M1) and Pan[13] (M2), this method has better accuracy.

When the Rayleigh damping coefficients are calculated using the method proposed in this paper (M4), the relative error of the peak value of displacement is $-3.8\% \sim 7.4\%$, that of velocity is $-4.5\% \sim 6.1\%$, and that of acceleration response is $-15.9\% \sim 7.6\%$. The error of displacement is basically within $\pm 5\%$ (except point A and point B under the excitation of CPM-030 seismic wave, and point D under the excitation of MLS-270 seismic wave), the error of velocity is basically within $\pm 5\%$ (except point D under the excitation of R1-10%, C08-050, FER-T1 and MLS-270 seismic waves), and the error of acceleration is basically within $\pm 10\%$ (except point C under the excitation of MCH-000 and S3-270 seismic waves, and point D under the excitation of MCH-000, S3-270 and R1-63% seismic waves).

In terms of the error range, the maximum absolute value of the relative error of the peak acceleration that is obtained by using the method proposed in this paper (M4) is slightly larger than

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that concluded by using M3 (the former is 15.9% and the latter is 14.8%), while, considering the error distribution in 0, the errors of acceleration are basically the same, but the errors of displacement and velocity are greatly amended.

From 0 and 0, it can be found that among the four methods adopted in this paper, M2 leads to the smallest error of the peak value of displacement (basically within the range of $\pm5\%$), and the smallest standard deviation of error which means the best applicability. If only the displacement of the site is concerned, it is recommended to use this method. The proposed method in this paper (M4) has the smallest error of the peak value of velocity (basically within the range of $\pm5\%$), and the smallest standard deviation of error which means the best applicability. This method is recommended if only the velocity of the site is concerned. M3 and the proposed method in this paper (M4) have the slightest error of the peak value of acceleration (basically within the range of $\pm10\%$), and the smallest standard deviation of error which means the best applicability. If only the acceleration of the site is concerned, these two methods are the recommended choices.

Taken together, when the displacement, velocity and acceleration are all considered in the analysis, the proposed method in this paper (M4) has the highest accuracy and best applicability (the displacement and velocity errors are basically within the range of $\pm5\%$ and the acceleration error is basically within the range of $\pm10\%$).

CONCLUSION

This paper establishes a dual parameter optimization theory for calculation of Rayleigh damping coefficients. Firstly, the functional relation between the relative error of dynamic response and Rayleigh damping coefficients is established based on CQC method. Secondly, by taking the square sum of the errors of peak value of displacement and the error of peak value of acceleration at the multiple points (DOF) of the surface of the complex site as the control objective, the equations for solving Rayleigh damping coefficients are obtained based on the principle of minimizing the control objective. Then, as an example, the seismic response of a valley under the excitation of 28 representative seismic waves which are randomly selected is calculated and the error due to Rayleigh damping model is analysed. The numerical result verifies the accuracy and applicability of the proposed method. Taken together, when the displacement, velocity and acceleration are all considered in the analysis, the proposed method in this paper has the highest accuracy and best applicability (the displacement and velocity errors are basically within the range of $\pm5\%$ and the acceleration error is basically within the range of $\pm10\%$).

Major engineering structures generally require seismic response analysis of the complex site. The fundamental frequency of site with deep deposit is often low, much lower than the frequency of the main component of most of seismic wave from bedrock. At this time, the proposed method in this paper can provide a good accuracy for calculation.

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