MULTI-OBJECTIVE OPTIMIZATION OF SUPPORTING PLAN FOR DEEP FOUNDATION USING ENTROPY-BASED UM-DEA

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ABSTRACT

In regard to optimize the supporting plan of deep foundation pit, this paper used the unascertained measurement (UM) and data envelopment analysis (DEA) to conduct. In order to determine the relationship between influencing factors and the best bid plan, an evaluation model for deep foundation pit support schemes based on UM was developed. First, the information entropy (IE) was introduced to determine the weight of discriminant indices which consider the confidence identification criteria as the judgment principle of evaluation. Then, the optimal solution from all feasible support schemes was investigated. Finally, Fuzzy comprehension assessment-data envelopment analysis (FCA-DEA) was utilized to analyse the effectiveness of design plans, which was evaluated by UME subsequently. Applicability of the proposed UM-DEA model was tested with four real design cases of foundation projects. Results compared with other methods have shown the developed model is useful for concept design and decision making of supporting plan for deep foundation.

KEYWORDS

Deep foundation pit, Unascertained measurement evaluation, Data envelopment analysis, Information entropy

INTRODUCTION

With the rapid development of civil engineering projects, deep foundation pit construction with large amounts of land and the complicated influence of surrounding environment is becoming much more common. Deep foundation pit construction is the key and basis of building construction. Its safety and reliability has direct relationship with high-rise building’s safety, stability and longevity. The successful level of any foundation construction projects can be argued to construct largely on selecting the most competent design plan. Accordingly, the selection of a design plan is one of the most important issues in construction projects. The complexity of foundation projects with comprehensive and uncertain conditions result in the process of design becoming a research topic which is full of high difficulty and high risk. The accident rate of the deep foundation pit engineering in high-rise buildings accidents accounted for nearly 10%. Both the economic losses and threat to safety of the workers are also serious [1,2]. Therefore, how to choose the best supporting system of deep foundation pit is closely related to the safety and economy of the whole project. In the design phase of deep foundation pit engineering, firstly, several kinds of feasibility supporting schemes are provided by experts [3]. Then, selecting the most appropriate plan for deep foundation pit is the most important issue. The optimization of each plan of the deep foundation pit supporting scheme will be evaluated and selected according to multiple targets [4,5].

To address those issues, this research rigorously identified the main influence factors affecting the selection process to incorporate criteria other than the factors inherited in foundation
project. The influential criteria were selected based on experts’ opinions and the ideas from the literature. And UM and entropy were utilized to address the uncertainty and interdependency among criteria [6,7]. The entropy is a powerful tool that can quantify the uncertainty inherited in respondents’ weighting of criteria to decrease the subjective and arbitrary decisions. Then, DEA [8,9] was introduced to evaluate the effectiveness of the plan selected and identified the relationship between criteria and results. Finally, a model which can select the most appropriate design support schemes for deep excavation was developed with using those tools. And the validity of the proposed model was tested with real cases.

METHODS

The developed methodology, as shown in Figure 1, starts with a brief literature review to illustrate various approaches utilized by researchers for design plan selection methods of deep foundation projects.

![Fig.1 - Research techniques](image)

The criteria needed were selected from two stages. The first stage was prepared from the literature requesting, thereafter, experts specialized in the safety, economy, environment, duration and quality to fill out a form that asked for a list of the most influential criteria in design plan selection. When the most important affected criteria of deep foundation design plan selection were identified, according to the collected data, relative weights of criteria were determined using the IE technique. The criteria weights obtained from IE analysis were then used as input for the proposed simulation model for deep foundation design plan selection based on UM. Finally, the developed
model was tested using case foundation projects, and an effective analysis was conducted to show the effectiveness of outputs to any changes in the inputs. This integration considered factors’ interdependency (using UM), made decisions under uncertainty (using simulation/UM), and handled decisions that involved large numbers of variables.

The uncertainty measure theory is different from fuzzy mathematics, grey theory and the random information, and it is a kind of new uncertainty theory[10,11].In the model, there are \( n \) evaluate samples that can be expressed using \( x_1, x_2, \ldots, x_n \), and the sample space can be used \( U = \{x_1, x_2, \ldots, x_n\} \) to express. Any object \( x_i \) consists of \( m \) evaluation indexes. And the index space can be used \( I = (I_1, I_2, \ldots, I_m) \) to express, when \( x_i \in U \), \( x_{ij} \) is the observed values \( I_j \) of \( j \) index in the sample \( x_i \), the evaluation space can be used \( L = (L_1, L_2, \ldots, L_k) \) to express, \( L_k = \) the \( k \) th grade, \( 1 \leq k \leq K \), where, \( K \) is grade of evaluation.

**Single-index uncertainty measure**

If there is a fixed object \( x_i \in U \) and an indicator \( I_j \), the observed value of \( x_i \) for \( I_j \) can be expressed by \( x_{ij} \), when the map \( \mu \) exists, \( A \) is the arbitrary set, and the object is subject to satisfy:

\[
\mu_{ijA} = \sum_{c=1}^{n} (x_{ij}) \\
0 \leq \mu_{ij}(x_{ij}) \leq 1 \\
\mu_{ij}(x_{ij}) = 1
\]

Where, \( i=1,2,\ldots, n \), \( j=1,2,\ldots,m \), the formula (1) is expressed "additive property "; the formula(2) is expressed "nonnegative bounded "; the formula (3) is expressed "polarity ". When the above three kinds of characteristics are satisfied, \( \mu_{ij}(x_{ij}) \) be called uncertainty measure function, namely measure.

According to the principle of the uncertainty measure, abiding by the law of the curve of the index, we construct a suitable single index measure function:

\[
\mu_i = \left[ \begin{array}{cccc}
\mu_{i11} & \mu_{i12} & \cdots & \mu_{i1k} \\
\mu_{i21} & \mu_{i22} & \cdots & \mu_{i2k} \\
\vdots & \vdots & \ddots & \vdots \\
\mu_{im1} & \mu_{im2} & \cdots & \mu_{imk}
\end{array} \right] = (\mu_{ijk})_{mn}\]

**Determination for weight of index**

The entropy method is a kind of objective weighting method, and the index weight can be calculated by the size of information through each indicator contained. The entropy method determines index weight by the relationship between the original data to decrease the subjective and arbitrary decisions, so the overall evaluation results are more objective [12].

When the measurement value \( x_{ij} \) and evaluation index are given, the measured value of the attribute is given, and each weight is calculated by the measured value. According to the theory of information entropy, the weight of index is determined:
In the formula, the size of $v_{ij}$ can reflect the importance or match the index $j$, then the weight of the index $j$ is defined as:

$$w_j = v_{ij} / \sum_{j=1}^m v_{ij}, \quad 0 \leq w_j \leq 1, \quad \sum_{j=1}^m w_j = 1$$

(6)

Multi index comprehensive measure matrix

The single-index uncertainty measure can be calculated by formula (4), and the weight vector of the index can be calculated by formula (5) and formula (6):

$$p = w_j \ast \left( \mu_{ijk} \right)_{mn}$$

(7)

Confidence criterion

Introducing "confidence" as the recognition criteria, according to the confidence threshold value $\lambda$ ($\lambda \geq 0.5$), usually in the range of $[0.6, 0.8]$.

$$k_{\lambda} = \min \left( k : \sum_{i=1}^k \mu_{ii} \geq \lambda, \quad 1 \leq k \leq K \right)$$

(8)

Then the evaluation results are considered as the $L_{k_\lambda}$ evaluation grades of $k_\lambda$.

Priority ranking

When $L_1 > L_2 > \cdots > L_k > \cdots > L_K$, the assignment of $L_i$ is $C_i$, and $C_i > C_{i+1}$, then:

$$Q_{k_i} = \sum_{i=1}^k C_i \mu_{ik}$$

(9)

$Q_{k_i}$ is the degree of uncertainty of the evaluation scheme $R_i$, $Q = \{Q_{k_1}, Q_{k_2}, \cdots, Q_{k_k}\}$ is called the vector of the degree of uncertainty, according to its superior ranking.

Effective analysis of design scheme based on FCA-DEA

Using the method of the evaluation of the uncertainty measure, a set of comprehensive measures evaluation can be ranked, and the optimal design plan could be obtained. However, when making a plan decision, only the pros and cons of each obtained design plan may not be sufficient, the following questions are more important. What is insufficient of each design scheme is that it cannot make sure whether can we turn it into effective suitable design plan, etc. This paper introduces the method of data envelopment analysis to deal with these problems.

If there are $n$ design plans for a project, and $p$ is used to represents any plan, $i$ indicates analysis index. The single-index uncertainty measure matrix is $\{\mu_{ijk}(p)\}_{mn}$, $\mu_{ijk}(p) \in [0, 1]$. 

$$v_{ij} = 1 + \frac{1}{\log k} \sum_{k=1}^K \mu_{ijk} \log \mu_{ijk}$$

(5)
Based on the theory of uncertainty measure,

\( \{ \mu_{ijk}^{(p)} \} / \sum_{j=1}^{c} \mu_{ijk}^{(p)}, \mu_{2k}^{(p)} / \sum_{j=1}^{c} \mu_{j(k)}^{(p)}, \ldots, \mu_{ijk}^{(p)} / \sum_{j=1}^{c} \mu_{ijk}^{(p)} \} \) is got for basic distribution information of all the results evaluated by the scheme \( P \) relative to the index \( i \). According to the information entropy, the total value \( Q_{i}^{p} \) of index \( i \) can be calculated as follows:

\[
Q_{i}^{p} = \frac{\sum_{j=1}^{c} \mu_{ijk}^{(p)} w_{j}}{\sum_{j=1}^{c} \mu_{j(k)}^{(p)}}
\]  

(10)

Where \( Q_{i}^{p} = \mu_{ijk}^{(p)} \) and \( Q^{p} = (Q_{1}^{p}, Q_{2}^{p}, \ldots, Q_{n}^{p}) \) indicates that the characteristics and status of each indicator of the plan \( p \). According to the production theory in DEA, the possible compositions of the production mainly contain three conditions: convex, non-effective and minimal. The set needed is:

\[
LDT = \{ y | y_{i} \leq \sum_{p=1}^{n} \frac{\sum_{j=1}^{c} \mu_{ijk}^{(p)} w_{j}}{\sum_{j=1}^{c} \mu_{j(k)}^{(p)}}, \ i = 1, 2, \ldots, n, \sum_{p=1}^{n} \lambda_{p} = 1 \}
\]  

(11)

According to formula (11), we can define supporting scheme FCA-DEA effectiveness. Three theorems are given as follows:

**Theorem 1:** The design scheme \( P^{0} \) for the FCA-DEA is valid when and only if

\[
\begin{bmatrix}
\min \left[ \varphi - \sum_{i=1}^{n} \frac{\sum_{j=1}^{c} \mu_{ijk}^{(p)} w_{j}}{\sum_{j=1}^{c} \mu_{j(k)}^{(p)}} U_{i} \right] = V_{F-D} \\
(F - D) \end{bmatrix}
\]

s.t. \[ \varphi - \sum_{i=1}^{n} \frac{\sum_{j=1}^{c} \mu_{ijk}^{(p)} w_{j}}{\sum_{j=1}^{c} \mu_{j(k)}^{(p)}} U_{i} \leq \varphi \]

\[ \sum_{i=1}^{n} U_{i} = 1, U_{i} \geq 0, i = 1, 2, \ldots, n \]

\[ p = 1, 2, \ldots, n \]

(12)

**Theorem 2:** The design scheme \( P^{0} \) for the FCA-DEA is valid when and only if the optimal value is: \( V_{F-D} = 0 \).
Theorem 3: If the plan $p_0$ does not satisfy the FCA-DEA, the optimal solution of $(F - DD)$ is $\lambda, s$, then the $Q_p^b + s$ must be able to achieve FCA-DEA effective.

Where, $(F - D)$ and $(F - DD)$ are the fuzzy decision of multi-objective optimization and the duality problem of $(F - D)$, respectively. From the above three theorems, we can construct the DEA model, carry out the scheme of FCA-DEA validity analysis.

RESULTS

Affected criteria of design plan selection

This project included the construction of an area of 8100m² underground car park with depth of 6.2m. In the direction of east, west and south, project site is adjacent to highway, office building is away from about 15m in the north direction, and there is an athlete field in the north-west direction. The influence of the building, road, underground lines and the plenum of the pipeline is affected by the settlement. Soil is composed of plain fill, fine sand, silt, fine sand, silty soil and there is strong weathered mudstone from top to bottom. Engineering geological conditions and characteristics are shown in Table 1.

<table>
<thead>
<tr>
<th>Soil classification</th>
<th>Soil characteristics</th>
<th>Soil thickness(m)</th>
<th>Soil gravity $\gamma$ (kN/m³)</th>
<th>Internal friction angle $\varphi(\degree)$</th>
<th>Cohesion C(kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain fill</td>
<td>Made up of sand, gravel, clay</td>
<td>2.5</td>
<td>17.2</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Fine sand</td>
<td>Containing 20% of silt, loose, saturated</td>
<td>2.5</td>
<td>17.8</td>
<td>30</td>
<td>6</td>
</tr>
<tr>
<td>Silt</td>
<td>Dark grey, plastic flow, saturated</td>
<td>4</td>
<td>16.8</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>Sand</td>
<td>Gray, little compaction saturated</td>
<td>2.6</td>
<td>19.0</td>
<td>30</td>
<td>6</td>
</tr>
<tr>
<td>Silty clay</td>
<td>Containing a small amount of powder sand, hard plastic</td>
<td>1.6</td>
<td>16.9</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

Tab. 1- Engineering geological condition and characteristics
According to the main characteristics of foundation pit engineering, there are some ideas which are suitable for the primary support scheme: (1) Dense rows of bored piles with reinforced concrete interior support + single row deep mixing pile cut-off wall; (2) Retaining structure with double row piles and single row deep mixing pile cut-off wall; (3) Spraying the anchor net supporting + single row deep layer mixing pile cut-off wall; (4) Arching cement soil + steel casing drilling pile space combination support.

Following the model testing methodology, the below main characteristics need to be considered: the influence factors of the surrounding environment, engineering geology and hydrogeology of the foundation pit of the engineering project, the safety, economy, environment, duration and quality, as shown in Figure 2, were determined based on the selected criteria and subcriteria.

According to relevant specification and documents [9], etc., index values of deep excavation were determined, as shown in Table 2, therefore, through formula (1)-(4), the main technical and economic index values were calculated, as shown in Table 3.

### Table 2 - Standard values of selection criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Excellent</th>
<th>Good</th>
<th>Poor</th>
<th>Very poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability of construction technology $C_1$</td>
<td>$\geq 1$</td>
<td>[0.85,0.95]</td>
<td>[0.75,0.85]</td>
<td>$\leq 0.75$</td>
</tr>
<tr>
<td>Coefficient of safety stability $C_2$</td>
<td>$\geq 2$</td>
<td>[1.7,1.9]</td>
<td>[1.5,1.7]</td>
<td>$\leq 1.5$</td>
</tr>
<tr>
<td>Difficulty of construction $C_3$</td>
<td>$\leq 0.7$</td>
<td>[0.7,0.8]</td>
<td>[0.8,0.9]</td>
<td>$\geq 1$</td>
</tr>
<tr>
<td>Reliability of design theory $C_4$</td>
<td>$\geq 1$</td>
<td>[0.85,0.95]</td>
<td>[0.75,0.85]</td>
<td>$\leq 0.75$</td>
</tr>
<tr>
<td>Comprehensive cost $C_5$</td>
<td>$\leq 200$</td>
<td>[200,225]</td>
<td>[225,275]</td>
<td>$\geq 300$</td>
</tr>
<tr>
<td>Surrounding buildings $C_6$</td>
<td>$\leq 0.7$</td>
<td>[0.7,0.8]</td>
<td>[0.8,0.9]</td>
<td>$\geq 1$</td>
</tr>
<tr>
<td>Surrounding distribution utilities and underground structures and utilities $C_7$</td>
<td>$\leq 0.7$</td>
<td>[0.7,0.8]</td>
<td>[0.8,0.9]</td>
<td>$\geq 1$</td>
</tr>
<tr>
<td>Surrounding road transportation $C_8$</td>
<td>$\leq 0.7$</td>
<td>[0.7,0.8]</td>
<td>[0.8,0.9]</td>
<td>$\geq 1$</td>
</tr>
<tr>
<td>Construction duration $C_9$</td>
<td>$\leq 30$</td>
<td>[30,40]</td>
<td>[40,60]</td>
<td>$\geq 70$</td>
</tr>
<tr>
<td>Maximum horizontal Displacement $C_{10}$</td>
<td>$\leq 20$</td>
<td>[20,25]</td>
<td>[25,35]</td>
<td>$\geq 40$</td>
</tr>
<tr>
<td>Effectiveness of precipitation $C_{11}$</td>
<td>$\geq 1$</td>
<td>[0.85,0.95]</td>
<td>[0.75,0.85]</td>
<td>$\leq 0.75$</td>
</tr>
<tr>
<td>Waterproof effects $C_{12}$</td>
<td>$\geq 1$</td>
<td>[0.85,0.95]</td>
<td>[0.75,0.85]</td>
<td>$\leq 0.75$</td>
</tr>
</tbody>
</table>

### Table 3 - Technical and economic values of schemes

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Excellent</th>
<th>Good</th>
<th>Poor</th>
<th>Very poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.90</td>
<td>1.00</td>
<td>0.80</td>
<td>0.95</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1.68</td>
<td>1.83</td>
<td>1.95</td>
<td>1.89</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.95</td>
<td>0.85</td>
<td>1.00</td>
<td>0.90</td>
</tr>
<tr>
<td>$C_4$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.70</td>
<td>0.90</td>
</tr>
<tr>
<td>$C_5$</td>
<td>335.10</td>
<td>283.70</td>
<td>252.30</td>
<td>223.90</td>
</tr>
<tr>
<td>$C_6$</td>
<td>0.95</td>
<td>0.85</td>
<td>0.95</td>
<td>0.80</td>
</tr>
<tr>
<td>$C_7$</td>
<td>0.95</td>
<td>0.85</td>
<td>0.90</td>
<td>0.80</td>
</tr>
<tr>
<td>$C_8$</td>
<td>0.95</td>
<td>0.80</td>
<td>0.95</td>
<td>0.80</td>
</tr>
<tr>
<td>$C_9$</td>
<td>65.00</td>
<td>60.00</td>
<td>70.00</td>
<td>40.00</td>
</tr>
<tr>
<td>$C_{10}$</td>
<td>47.00</td>
<td>26.00</td>
<td>29.00</td>
<td>34.00</td>
</tr>
<tr>
<td>$C_{11}$</td>
<td>0.90</td>
<td>0.87</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>0.93</td>
<td>0.92</td>
<td>0.90</td>
<td>0.80</td>
</tr>
</tbody>
</table>
Determining single-index measure matrix

Through the classification of all the individual indicators, the evaluation level can be considered as excellent, good, poor and very poor. The main technical and economic indexes of each plan were obtained through the investigation of experts, and the single-index of the four kinds of supporting schemes was calculated as follows:

\[
(\mu_{1jk})_{12\times4} = \begin{pmatrix}
0 & 0.5 & 0.5 & 0 \\
0 & 0 & 0.9 & 0.1 \\
0 & 0 & 0.5 & 0.5 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0.5 & 0.5 \\
0 & 0 & 0.5 & 0.5 \\
0 & 0 & 0.5 & 0.5 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0.5 & 0.5 \\
0 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0.5 & 0 \\
0 & 0 & 0.2 & 0
\end{pmatrix}
\]

\[
(\mu_{2jk})_{12\times4} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0.65 & 0.35 \\
0 & 0 & 0.5 & 0.5 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0.652 & 0.348 \\
0 & 0 & 0.5 & 0.5 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0.9 & 0.1 & 0 \\
0 & 0.2 & 0.8 & 0 \\
0 & 0.7 & 0.3 & 0
\end{pmatrix}
\]
Where, \((\mu_{ijk})_{12\times4}\), \((\mu_{2jk})_{12\times4}\), \((\mu_{3jk})_{12\times4}\) and \((\mu_{4jk})_{12\times4}\) indicate the single-index unascertained measure matrices of scheme 1, scheme 2, scheme 3 and scheme 4 respectively.

**Weight\((W_i)\) determination for plan optimization**

According to formula (5) and formula (6), the weight vectors of the four programs were determined respectively:

\[
W_1 = (0.0632 0.0957 0.0632 0.127 0.127 0.0632 0.0632 0.127 0.127 0.0632 0.0632) \\
W_2 = (0.1172 0.0626 0.0586 0.1172 0.0626 0.0586 0.0586 0.1172 0.1172 0.0897 0.0749) \\
W_3 = (0.0587 0.0587 0.1174 0.1174 0.0591 0.0587 0.1174 0.0587 0.1174 0.0604 0.1174) \\
W_4 = (0.0987 0.0846 0.0987 0.0493 0.0859 0.0859 0.0987 0.0987 0.0987 0.0987 0.0754)
\]

According to the formula (7), the final comprehensive measure of the evaluation vectors were obtained respectively as in Table 4.

The confidence degree is 0.6, based on formula (8), four evaluation grades of deep foundation pit supporting scheme of the comprehensive evaluation results of the uncertainty measure were given. In Table 4, it can be concluded that the results of four optimal deep foundation pit support schemes are “poor, good, general and good”. According to the principle of superiority ranking, the superiority of all schemes is calculated as (1.9603 2.8344 1.8579 2.9928), therefore the superiority ranking is scheme 4 > scheme 2 > scheme 1 > scheme 3. The scheme 4 of arch cement soil + type steel drilling pile space combination supporting is the optimal supporting structure. The results are the same as actual supporting structure scheme. And the evaluation results of the grey fuzzy variable decision model in Table 5, were compared and analysed. The optimal results are basically in line with the feasibility.

**Tab. 4 - Results of the uncertainty measurement evaluation**

<table>
<thead>
<tr>
<th>Scheme No.</th>
<th>Synthetic uncertainty measure</th>
<th>Discriminate result</th>
<th>Priority ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Excellent</td>
<td>Good</td>
<td>General</td>
</tr>
<tr>
<td>1</td>
<td>0.127</td>
<td>0.1279</td>
<td>0.3235</td>
</tr>
<tr>
<td>2</td>
<td>0.2344</td>
<td>0.3874</td>
<td>0.3564</td>
</tr>
<tr>
<td>3</td>
<td>0.0294</td>
<td>0.1805</td>
<td>0.4086</td>
</tr>
<tr>
<td>4</td>
<td>0.0038</td>
<td>0.6882</td>
<td>0.2705</td>
</tr>
</tbody>
</table>

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Table 5 - Results of grey-fuzzy variable evaluation

<table>
<thead>
<tr>
<th>Scheme No.</th>
<th>Scheme 1</th>
<th>Scheme 2</th>
<th>Scheme 3</th>
<th>Scheme 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1$</td>
<td>$\alpha = 1$</td>
<td>$\alpha = 2$</td>
<td>$\alpha = 2$</td>
<td>$\alpha = 1$</td>
</tr>
<tr>
<td>$p = 1$</td>
<td>$p = 2$</td>
<td>$p = 1$</td>
<td>$p = 2$</td>
<td>$p = 1$</td>
</tr>
<tr>
<td>1</td>
<td>0.410</td>
<td>0.373</td>
<td>0.327</td>
<td>0.261</td>
</tr>
<tr>
<td>2</td>
<td>0.709</td>
<td>0.601</td>
<td>0.856</td>
<td>0.694</td>
</tr>
<tr>
<td>3</td>
<td>0.497</td>
<td>0.563</td>
<td>0.493</td>
<td>0.624</td>
</tr>
<tr>
<td>4</td>
<td>0.864</td>
<td>0.833</td>
<td>0.976</td>
<td>0.961</td>
</tr>
</tbody>
</table>

Effectiveness analysis

The values $Q_i^p$ is $R = (R_1, R_2, R_3, R_4, R_5)$, and the five indicators respectively correspond to the safety, economy, environment, construction period and quality. Then the $F-\text{DD}$ models were calculated, the results can be shown in Table 6.

Table 6 - Values of criteria $s$ and the optimal value $V_{F-\text{DD}}$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Scheme 1</th>
<th>Scheme 2</th>
<th>Scheme 3</th>
<th>Scheme 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0.1631</td>
<td>0.2021</td>
<td>0.3044</td>
<td>0.2573</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0.2542</td>
<td>0.1255</td>
<td>0.0871</td>
<td>0</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0.1854</td>
<td>0.0848</td>
<td>0.1263</td>
<td>0.0575</td>
</tr>
<tr>
<td>$s_4$</td>
<td>0.2045</td>
<td>0.2064</td>
<td>0.3034</td>
<td>0</td>
</tr>
<tr>
<td>$s_5$</td>
<td>0.1437</td>
<td>0</td>
<td>0.0956</td>
<td>0.1078</td>
</tr>
<tr>
<td>$V_{F-\text{DD}}$</td>
<td>0.9509</td>
<td>0.6188</td>
<td>0.9168</td>
<td>0.4226</td>
</tr>
</tbody>
</table>

From Table 6, it can be concluded that the optimal value $V_{F-\text{DD}}$ of scheme 4 is minimum, if the size of $s_1, s_2$ and $s_3$ can be adjusted, reaching the state of $s_1 = 0, s_2 = 0$ and $s_3 = 0$. Then scheme 4 can achieve the effectiveness of FCA-DEA. In addition, comparing several kinds of supporting scheme to arched soil cement + steel drilling pile space combination support the supporting scheme, it is not possible to be more effective than scheme 4, but it can be helpful to improve some indicators of performance.

In the feasibility research of the support scheme, the application of double row pile retaining + single row deep mixing pile anti-seepage curtain wall supporting scheme (scheme 2) can be clearly shown. Through the above analysis, it is proved that scheme 2 of unascertained measure evaluation has its advantages compared to scheme 4. So it also has reference to other scheme contains information, analysis of the deficiencies, and to explore whether there can be further improved. According to Table 6, double row piles supporting + single row deep mixing pile cut off curtain wall of a program $s_1, s_2, s_3, s_4$ and $s_5$ are not 0. That is to say that the indicators of safety, economy, environment and time limit for a project have not reached effectively compared to FCA-DEA, it needs to be improved. If the security, economic, environmental, duration and other indicators of the scheme 2 are increased

$$(Q_1^p + s_1, Q_2^p + s_2, Q_3^p + s_3, Q_4^p + s_4) = (0.2943, 0.6233, 0.3747, 0.4364)$$

Then, the scheme 2 can achieve the effective state of FCA-DEA.
CONCLUSION

It is vital to choose reasonable supporting scheme for deep foundation pit, the paper used unascertained measurement principle and data envelopment analysis theory to optimize the supporting scheme. The following conclusions can be drawn:

(1) Evaluation of supporting schemes for deep foundation pit is generally subject to a number of evaluation indices to determine in the process of the evaluation indicators of uncertainty, the aspects of safety, environment and quality are considered with using the unascertained measure theory. And the construction of deep foundation pit supporting scheme optimization of unascertained measure evaluation model can be established;

(2) The use of information entropy theory can determine the index weight to avoid subjective random;

(3) The DEA model was used to analyse deep foundation design plans, showing the validity of FCA-DEA in the deep foundation pit.

REFERENCES